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Invariant approximations for commuting mappings in CAT(0) and hyperconvex spaces

Naseer Shahzad [a], Jack Markin [b,*]

[a] *Department of Mathematics, King Abdul Aziz University, PO Box 80203, Jeddah 21589, Saudi Arabia*
[b] *528 Rover Blvd., Los Alamos, NM 87544, USA*

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Abstract

In this paper, for a commuting pair consisting of a point-valued nonexpansive self-mapping t and a set-valued nonexpansive self-mapping T of a hyperconvex metric space (or a CAT(0) space) X, we look for a solution to the problem of existence of $z \in E \subset X$ such that

$$d(z, y) = d(y, E) \quad \text{and} \quad z = t(z) \in T(z).$$

Keywords: Commuting mappings; Nonexpansive mappings; Hyperconvex spaces; CAT(0) space; Fixed point

1. Introduction

Given a subset E of a metric space X, the set of best approximations to $y \in X$ from $E \subset X$ is defined by $P_E(y) = \{z \in E\colon d(z, y) = d(y, E)\}$, where $d(y, E) = \inf\{d(y, x)\colon x \in E\}$. The set E is proximinal if $P_E(y)$ is nonempty for each $y \in X$ and Chebyshev if $P_E(y)$ is a singleton for each $y \in X$.

In this paper, for a pair consisting of a point-valued self-mapping t and a set-valued self-mapping T of a hyperconvex metric space (or a CAT(0) space) X, we look for a point $z \in P_E(y)$ such that $z = t(z) \in T(z)$; in other words, we look for a solution to the problem of existence of $z \in E$ such that

$$d(z, y) = d(y, E) \quad \text{and} \quad z = t(z) \in T(z). \tag{1}$$

Meinardus [1] was the first who studied the existence of such a solution in the space of all continuous real-valued functions with sup-norm for point-valued nonexpansive mappings. Subrahmanyam [2] generalized the Meinardus result as follows.

* Corresponding author.
E-mail address: jmarkin@newmexico.com (J. Markin).

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