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ON LEFT (θ,ϕ)-DERIVATIONS OF PRIME RINGS

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Abstract. Let R be a 2-torsion free prime ring. Suppose that θ,ϕ are automorphisms of R. In the present paper it is established that if R admits a nonzero Jordan left (θ,θ)-derivation, then R is commutative. Further, as an application of this resul it is shown that every Jordan left (θ,θ)-derivation on R is a left (θ,θ)-derivation on R. Finally, in case of an arbitrary prime ring it is proved that if R admits a left (θ,ϕ)-derivation which acts also as a homomorphism (resp. anti-homomorphism) on a nonzero ideal of R, then $d=0$ on R.

1. Introduction

Throughout the present paper R will denote an associative ring with centre $Z(R)$. Recall that R is prime if $aRb=\{0\}$ implies that $a=0$ or $b=0$. As usual $[x,y]$ will denote the commutator $xy-yx$. An additive subgroup U of R is said to be a Lie ideal of R if $[u,r]\in U$ for all $u\in U$, $r\in R$. Suppose that θ,ϕ are endomorphisms of R. An additive mapping $d: R\longrightarrow R$ is called a (θ,ϕ)-derivation (resp. Jordan (θ,ϕ)-derivation) if $d(xy)=d(x)\phi(y)+\theta(x)d(y)$, (resp. $d(x^2)=d(x)\phi(x)+\theta(x)d(x)$) holds for all $x,y\in R$. Of course, every $(1,1)$-derivation (resp. Jordan $(1,1)$- derivation), where 1 is the identity mapping on R is a derivation (resp. Jordan derivation) on R. An additive mapping $d: R\to R$ is called a left (θ,ϕ)-derivation (resp. Jordan left (θ,ϕ)-derivation) if $d(xy)=\theta(x)d(y)+\phi(y)d(x)$ (resp. $d(x^2)=\theta(x)d(x)+\phi(x)d(x)$) holds for all $x,y\in R$. Clearly, every left $(1,1)$-derivation (resp. Jordan left $(1,1)$-derivation) is a left derivation (resp. Jordan left derivation) on R. Obviously, every left derivation is a Jordan left derivation but the converse need not be true in general. Recently the author together with Nadeem [1] proved that the converse statement is true in the case when the underlying ring is prime and 2-torsion free. In the present paper we shall show that if a 2-torsion free prime ring R admits an additive mapping satisfying $d(u^2)=2\theta(u)d(u)$ for all $u\in U$, then either $d(U)=\{0\}$ or $U\subseteq Z(R)$ where U is a Lie ideal of R with $u^2\in U$ for all $u\in U$ and θ is an automorphism

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