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## Stresses in rotating heterogeneous viscoelastic composite cylinders with variable thickness\*

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Abstract An analytical solution is presented for the rotation problem of a two-layer composite elastic cylinder under a plane strain assumption. The external cylinder has variable-thickness formulation, and is made of a heterogeneous orthotropic material. It contains a fiber-reinforced viscoelastic homogeneous isotropic solid core of uniform thickness. The thickness and elastic properties of the external cylinder are taken as power functions of the radial direction. By the boundary and continuity conditions, the radial displacement and stresses for the rotating composite cylinder are determined. The effective moduli and Illyushin's approximation methods are used to obtain the viscoelastic solution to the problem. The e ects of heterogeneity, thickness variation, constitutive, time parameters on the radial displacement, and stresses are investigated.

Key words rotating composite cylinder, orthotropic, heterogeneous, variable-thickness, fiber-reinforced viscoelastic core

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## 1 Introduction

Rotating fiber-reinforced composite cylinders have gained increasing attention in recent years. This is due to their widespread use in mechanical engineering, such as high-speed gears, flywheels, turbine motors, and shrink fits. The analytical solution for the elastic problem of a uniform-thickness cylinder with homogeneous isotropic materials could be easily found in most standard elasticity books<sup>[1]</sup>. However, relatively little about the problem for inhomogeneous anisotropic materials could be found in the literatures<sup>[2–3]</sup>. Liew et al.<sup>[4]</sup> studied the thermal stress of functionally graded hollow circular cylinders. Oral and Anlas<sup>[5]</sup> analyzed the e ect of continuous inhomogeneity on the stress distribution for anisotropic cylindrical bodies. The stresses in exponentially varying property thick-walled cylinders under internal pressure were presented by Tutuncu<sup>[6]</sup>. Chandrashekhara and Gopalakrishnan<sup>[7]</sup> established the elastic analysis for a long two-layered circular cylinder having a thin orthotropic shell and a thick transversely isotropic core subjected to the axisymmetric load. Zenkour<sup>[8]</sup> obtained the stresses in a rotating variable-thickness orthotropic cylinder containing a solid core with uniform thickness.

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