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ON NONLOCAL FRACTIONAL BOUNDARY VALUE PROBLEMS

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Abstract. This paper studies a new class of nonlocal boundary value problems of nonlinear differential equations of fractional order. We extend the idea of a three-point nonlocal boundary condition ($x(1) = \alpha x(\eta),\ \alpha \in R,\ 0 < \eta < 1$) to a nonlocal strip condition of the form: $x(1) = \eta \int_{\nu}^{\tau} x(s)ds,\ 0 < \nu < \tau < 1$. In fact, this strip condition corresponds to a continuous distribution of the values of the unknown function on an arbitrary finite segment of the interval. In the limit $\nu \to 0,\ \tau \to 1$, this strip condition takes the form of a typical integral boundary condition. Some new existence and uniqueness results are obtained for this class of nonlocal problems by using standard fixed point theorems and Leray-Schauder degree theory. Some illustrative examples are also discussed.

Keywords. Fractional differential equations; nonlocal boundary conditions; fixed point theorems; Leray-Schauder degree.

AMS (MOS) subject classification: 26A33, 34A12, 34A40.

1 Introduction

In recent years, boundary value problems of nonlinear fractional differential equations have been addressed by several researchers. Fractional derivatives provide an excellent tool for the description of memory and hereditary properties of various materials and processes, see [17]. These characteristics of the fractional derivatives make the fractional-order models more realistic and practical than the classical integer-order models. As a matter of fact, fractional differential equations arise in many engineering and scientific disciplines such as physics, chemistry, biology, economics, control theory, signal and image processing, biophysics, blood flow phenomena, aerodynamics, fitting of experimental data, etc. [15, 18, 19]. For some recent development on the topic, see [1-8, 12, 13] and the references therein.

In this paper, we discuss the existence and uniqueness of solutions for a boundary value problem of nonlinear fractional differential equations of order