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Symmetrical Component Representation of a Six-Phase Salient-Pole Machine

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ABSTRACT. This paper presents a model of a 6-phase machine based on symmetrical component theory. A mathematical model of the machine, with nine windings, is developed into instantaneous symmetrical components. These, in turn, are used to derive expressions relating phasor symmetrical components of voltages and currents. Such a model is very useful in treating asymmetrical faults.

1. Introduction

In 1918, Fortescue presented a powerful tool for the analysis of an unbalanced n-phase system; that is the symmetrical component theory. Since then, a lot of work has been carried out to analyze various systems using this theory. The basic requirement of the theory is that circuits constituting the system are to be symmetrical. If this is not the case, the transformation to symmetrical components is of no use since the positive, negative and zero sequence circuits will have mutual coupling between them. As for the power systems, the various phases are not totally symmetrical and therefore, the use of symmetrical components may lead to inaccurate results as was found by some authors[1,2]. However, the synchronous machine is inherently symmetrical[3] and symmetrical components can, therefore, be used in the steady-state fault analysis.

Most of the researchers dealing with fault analysis of synchronous machines handle the problem using phasor symmetrical components[4,5,6]. This is done based on the physical picture since any set of unbalanced voltages (or currents) could be analyzed into three balanced sets; namely positive, negative and zero sequence components.

There has been an increasing interest in the 6-phase machine over the last decade[7,8,9]. It is the author's interest to study the behaviour of the 6-phase machine under various fault conditions. The aim of this paper is to develop simplified phasor

V-I relationships of the 6-phase machine starting from basic principles. It will be shown that the 6-phase machine terminal voltages and currents could be related to each other as phasor symmetrical components. Such result could be used by designers to analyze unbalanced faults of the 6-phase machine.

2. Mathematical Model of the Machine

The instantaneous voltage equations of the machine referred to one of the stator windings can be written in the following compact matrix form[9]

$$\begin{bmatrix} v_s \\ v_r \end{bmatrix} = \begin{bmatrix} -Z_{ss} & Z_{sr} \\ -Z_{rs} & Z_{rr} \end{bmatrix} \cdot \begin{bmatrix} i_s \\ i_r \end{bmatrix} \tag{1}$$

where

$[v_s]$ = the stator voltage matrix = $[v_a\ v_b\ v_c\ v_d\ v_e\ v_f]^T$

$[v_r]$ = the rotor voltage matrix referred to the stator = $[v_{fd}, v_{kd}, v_{kq}]^T = [v_{fd}, 0, 0]^T$

$[i_s]$ = the stator current matrix = $[i_a\ i_b\ i_c\ i_d\ i_e\ i_f]^T$

$[i_r]$ = the rotor current matrix referred to the stator = $[i_{fd}, i_{kd}, i_{kq}]^T$

$[Z_{ss}]$ = the stator self-impedance matrix

$$[Z_{ss}] = \begin{bmatrix} r_s + pL_{AA} & pL_{AB} & pL_{AC} & pL_{AD} & pL_{AE} & pL_{AF} \\ pL_{AB} & r_s + pL_{BB} & pL_{BC} & pL_{BD} & pL_{BE} & pL_{BF} \\ pL_{AC} & pL_{BC} & r_s + pL_{CC} & pL_{CD} & pL_{CE} & pL_{CF} \\ pL_{AD} & pL_{BD} & pL_{CD} & r_s + L_{DD} & pL_{DE} & pL_{DF} \\ pL_{AE} & pL_{BE} & pL_{CE} & pL_{DE} & r_s + pL_{EE} & pL_{EF} \\ pL_{AF} & pL_{BF} & pL_{CF} & pL_{DF} & pL_{EF} & r_s + pL_{FF} \end{bmatrix}$$

$[Z_{rs}] = [z_{sr}]^T$ = stator-to-rotor mutual impedance matrix

$$[Z_{rs}] = p \begin{bmatrix} L_{Afd} & L_{Bfd} & L_{Cfd} & L_{Dfd} & L_{Efd} & L_{Ffd} \\ L_{Akd} & L_{Bkd} & L_{Ckd} & L_{Dkd} & L_{Ekd} & L_{Fkd} \\ L_{Akq} & L_{Bkq} & L_{Ckq} & L_{Dkq} & L_{Ekq} & L_{Fkq} \end{bmatrix}$$

$[Z_{rr}]$ = the rotor self impedance matrix

$$[Z_{rr}] = \begin{bmatrix} r_{fd} + pL_{fdfd} & pL_{fdkd} & 0 \\ pL_{fdkd} & r_{kd} + pL_{kdkd} & 0 \\ 0 & 0 & r_{kq} + pL_{kqkq} \end{bmatrix}$$

The various elements of the impedance matrices are defined in the appendix.

In equation (1), it should be pointed out that the convention adopted for the signs of voltages is that $[v_s]$ and $[v_r]$ are the applied voltages at the terminals. The positive direction of stator currents $[i_s]$ corresponds to generation, while that of rotor currents $[i_r]$ corresponds to motor notation.

3. The Symmetrical Components Transformation

The impedance matrix in equation (1) has (77) terms most of which are functions of the rotor position making them non-linear. The explicit solution of the equations is therefore very difficult if not impossible. However, these equations may be simplified by applying the power invariance symmetrical components transformation matrix $[\phi_s]$ to the instantaneous stator voltage and current vectors. The transformation matrix $[\phi_s]$ is given by

$$[\phi_s] = \frac{1}{\sqrt{6}} \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & b & b^2 & -1 & b^4 & b^5 \\ 1 & b^2 & b^4 & 1 & b^2 & b^4 \\ 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & b^4 & b^2 & 1 & b^4 & b^2 \\ 1 & b^5 & b^4 & -1 & b^2 & b \end{bmatrix} \tag{2}$$

where

$$b = e^{\frac{i2\pi}{6}}$$

The instantaneous symmetrical components of the stator voltage $[v_s']$ and current $[i_s']$ vectors can thus be obtained as follows

$$\left.\begin{aligned} [v_s'] &= [\phi_s] \quad [v_s] \\ [i_s'] &= [\phi_s] \quad [i_s] \end{aligned}\right\} \tag{3}$$

It is worthy to mention that each rotor circuit has been dealt with as a separate single phase winding. Therefore, no transformation has been applied to the rotor variables. Then voltage equations (1) become

$$\begin{bmatrix} v_s' \\ v_r \end{bmatrix} = \begin{bmatrix} Z_{ss}' & Z_{sr}' \\ Z_{rs}' & Z_{rr}' \end{bmatrix} \begin{bmatrix} i_s' \\ i_r \end{bmatrix} \tag{4}$$

Where

$$[Z'_{ss}] = -[\phi_s]\ [Z_{ss}]\ [\phi_s]^{-1} =$$

$$\begin{bmatrix} -r_s - pl_s & 0 & 0 & 0 & 0 & 0 \\ 0 & -r_s - p\,(l_s + \frac{6}{2}L_1) & 0 & 0 & 0 & -p\frac{6}{2}L_2\,e^{j2\theta} \\ 0 & 0 & -r_s - pl_s & 0 & 0 & 0 \\ 0 & 0 & 0 & -r_s - pl_s & 0 & 0 \\ 0 & 0 & 0 & 0 & -r_s - pl_s & 0 \\ 0 & -p\frac{6}{2}L_2\,e^{-j2\theta} & 0 & 0 & 0 & -r_s - p\,(l_s + \frac{6}{2}L_1) \end{bmatrix} \quad (5-a)$$

$$[Z'_{sr}] = [\phi_s]\,[Z_{sr}] =$$

$$\begin{bmatrix} 0 & 0 & 0 \\ p\frac{\sqrt{6}}{2}(L_1 + L_2)\,e^{j\theta} & p\frac{\sqrt{6}}{2}(L_1 + L_2)\,e^{j\theta} & jp\frac{\sqrt{6}}{2}(L_1 - L_2)\,e^{j\theta} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ p\frac{\sqrt{6}}{2}(L_1 + L_2)\,e^{-j\theta} & p\frac{\sqrt{6}}{2}(L_1 + L_2)\,e^{-j\theta} & -jp\frac{\sqrt{6}}{2}(L_1 - L_2)\,e^{-j\theta} \end{bmatrix} \quad (5-b)$$

$$[Z'_{rs}] = -[Z_{rs}]\,[\phi_s]^{-1} = -[Z'_{sr}]^*_t =$$

$$\begin{bmatrix} 0 & -p\frac{\sqrt{6}}{2}(L_1 + L_2)\,e^{-j\theta} & 0 & 0 & 0 & -p\frac{\sqrt{6}}{2}(L_1 + L_2)\,e^{j\theta} \\ 0 & -p\frac{\sqrt{6}}{2}(L_1 + L_2)\,e^{-j\theta} & 0 & 0 & 0 & -p\frac{\sqrt{6}}{2}(L_1 + L_2)\,e^{j\theta} \\ 0 & jp\frac{\sqrt{6}}{2}(L_1 - L_2)\,e^{-j\theta} & 0 & 0 & 0 & -jp\frac{\sqrt{6}}{2}(L_1 - L_2)\,e^{j\theta} \end{bmatrix} \quad (5-c)$$

$$[Z'_{rr}] = [Z_{rr}] =$$

$$\begin{bmatrix} r_{fd} + p\,(l_{fd} + L_1 + L_2) & p\,(L_1 + L_2) & 0 \\ p\,(L_1 + L_2) & r_{kd} + p\,(l_{kd} + L_1 + L_2) & 0 \\ 0 & 0 & r_{kq} + p\,(l_{kq} + L_1 - L_2) \end{bmatrix} \quad (5-d)$$

$$
\begin{bmatrix} v_1 \\ v_p \\ v_3 \\ v_4 \\ v_5 \\ v_n \\ v_{fd} \\ 0 \\ 0 \end{bmatrix}
=
\begin{bmatrix}
-pl_s & & & & & & & & \\
 & -p(l_s+(6/2)L_1) & & & & -p(6/2)L_2 e^{j2\theta} & p(\sqrt{6}/2)(L_1+L_2)e^{j\theta} & p(\sqrt{6}/2)(L_1+L_2)e^{j\theta} & jp(\sqrt{6}/2)(L_1-L_2)e^{j\theta} \\
 & & -pl_s & & & & & & \\
 & & & -pl_s & & & & & \\
 & & & & -pl_s & & & & \\
 & -p(6/2)L_2 e^{-j2\theta} & & & & -p(l_s+(6/2)L_1) & p(\sqrt{6}/2)(L_1+L_2)e^{-j\theta} & p(\sqrt{6}/2)(L_1+L_2)e^{-j\theta} & -jp(\sqrt{6}/2)(L_1-L_2)e^{-j\theta} \\
 & -p(\sqrt{6}/2)(L_1+L_2)e^{-j\theta} & & & & -p(\sqrt{6}/2)(L_1+L_2)e^{j\theta} & p(l_{fd}+L_1+L_2) & p(L_1+L_2) & \\
 & -p(\sqrt{6}/2)(L_1+L_2)e^{-j\theta} & & & & -p(\sqrt{6}/2)(L_1+L_2)e^{j\theta} & p(L_1+L_2) & p(l_{kd}+L_1+L_2) & \\
 & jp(\sqrt{6}/2)(L_1-L_2)e^{-j\theta} & & & & -jp(\sqrt{6}/2)(L_1-L_2)e^{j\theta} & & & p(l_{kq}+L_1-L_2)
\end{bmatrix}
\begin{bmatrix} i_1 \\ i_p \\ i_3 \\ i_4 \\ i_5 \\ i_n \\ i_{fd} \\ i_{kd} \\ i_{kq} \end{bmatrix}
\quad (6)
$$

Substituting from equations (3) and (5) into equation (4), the machine voltage-current equations become as shown in equation (6). In equation (6) resistances have been ignored. This is the case since this equation is to be used for short circuit analysis where the effect is mainly due to inductances.

By inspecting equation (6), it is noted that the impedance matrix terms are decreased to 25 terms and the stator windings of the 6-phase salient-pole machine are replaced by the following two systems :

a) Four independent windings for the stator. The inductance of each winding is equal to the stator phase leakage inductance.

b) A salient-pole machine having two windings on the stator. The mutual coupling between the machine windings appear in complex form as indicated by equation (6).

From the last three equations of (6), one can obtain i_{kq}, i_{kd} and i_{fd} in terms of i_P, i_n and v_{fd}. Substituting these in the second and sixth equations of (6), it is found that

$$\begin{bmatrix} v_P \\ v_n \end{bmatrix} = \begin{bmatrix} e^{j\theta} \\ e^{-j\theta} \end{bmatrix} P_1 V_{fd} + \begin{bmatrix} P_2 & P_3 e^{j2\theta} \\ P_3 e^{-j2\theta} & P_2 \end{bmatrix} \begin{bmatrix} i_P \\ i_n \end{bmatrix} \tag{7}$$

Where

$\theta = \omega t + \phi$, ϕ being the angle between the d-axis and phase-a at $t = 0$

$$P_1 = \frac{\sqrt{6}}{2} \left(\frac{L_{md}\, l_{kd}}{l_{fd}\, l_{kd} + l_{fd}\, L_{md} + l_{kd}\, L_{md}} \right)$$

$$P_2 = - Pl_s - p\,\frac{3}{2} \left[\frac{L_{mq}\, l_{kq}}{L_{mq} + l_{kq}} + \frac{L_{md}\, l_{fd}\, l_{kd}}{L_{md}\, l_{fd} + L_{md}\, l_{kd} + l_{fd}\, l_{kd}} \right] \tag{8}$$

$$P_3 = - p\,\frac{3}{2} \left[- \frac{l_{kq}\, L_{mq}}{L_{kq} + l_{mq}} + \frac{l_{fd}\, l_{kd}\, L_{md}}{l_{fd}\, l_{kd} + l_{fd}\, L_{md} + l_{kd}\, L_{md}} \right]$$

$$L_{md} = L_1 + L_2 \quad , \quad L_{mq} = L_1 - L_2$$

It might be pointed out that, as the stator currents and voltages are instantaneous values, (*i.e.*, real values), their positive and negative symmetrical components (*i.e.*, V_p and V_n) are conjugates. This is a direct result from the nature of the symmetrical component transformation matrix $[\phi_s]$.

Let the 6-phase currents be

$$\left.\begin{aligned} i_a &= \sqrt{2}\, I_a \cos(\omega t + \theta_{ia}), & i_b &= \sqrt{2}\, I_b \cos(\omega t + \theta_{ib}) \\ i_c &= \sqrt{2}\, I_c \cos(\omega t + \theta_{ic}), & i_d &= \sqrt{2}\, I_d \cos(\omega t + \theta_{id}) \\ i_e &= \sqrt{2}\, I_e \cos(\omega t + \theta_{ie}), & i_f &= \sqrt{2}\, I_f \cos(\omega t + \theta_{if}) \end{aligned}\right\} \tag{9}$$

The above phase currents could be manipulated in a manner similar to the procedure given in reference [10] to get

$$i_p = \frac{1}{\sqrt{2}} e^{j\omega t} I_p + \frac{1}{\sqrt{2}} e^{-j\omega t} I_n^* \tag{10}$$

$$i_n = \frac{1}{\sqrt{2}} e^{-j\omega t} I_p^* + \frac{1}{\sqrt{2}} e^{j\omega t} I_n \tag{11}$$

where

I_p : Phasor positive-sequence current.

I_n : Phasor negative-sequence current.

Equations (10, 11) could be substituted into (7) to get

$$\begin{aligned} v_p &= e^{j(\omega t + \phi)} P_1 V_{fd} + \frac{P_2}{\sqrt{2}} (e^{j\omega t} I_p + e^{-j\omega t} I_n') \\ &+ \frac{P_{3e}{}^{j2\phi}}{\sqrt{2}} (e^{j\omega t} I_p^* + e^{j3\omega t} I_n) \\ &= (e^{j\phi} P_1 V_{fd} + \frac{P_2}{\sqrt{2}} I_p + \frac{P_3}{\sqrt{2}} e^{j2\phi} I_p^*) e^{j\omega t} + (\frac{P_2}{\sqrt{2}} I_n^*) e^{-j\omega t} \\ &+ (\frac{p_3}{\sqrt{2}} e^{j2\phi} I_n) e^{j3\omega t} \end{aligned} \tag{12}$$

From equation (12), it is seen that the instantaneous positive sequence voltage could be written as the sum of positive sequence, negative sequence and third harmonic negative sequence voltages. Thus

$$v_p = \frac{1}{\sqrt{2}} e^{j\omega t} V_p + \frac{1}{\sqrt{2}} e^{-j\omega t} V_n^* + \frac{1}{\sqrt{2}} e^{j3\omega t} V_{3-n} \tag{13}$$

It must be noted here that the assumption of fundamental frequency currents lead to fundamental and third harmonic voltage components. This is the case from the machine side. However, the external impedance relates voltages and currents. Thus, harmonic voltages give rise to harmonic currents which will again produce more harmonics. Therefore, ideally, voltages and currents must be expressed as the sum of infinite series of harmonics. For steady-state analysis, it is a common practice to ignore harmonics[4,5]. This is based on the fact that synchronous generators are designed such that harmonics are minimized[4]. Thus, the third harmonic component of the voltage could be ignored for steady-state analysis and, therefore, sequence reactances could be developed considering the fundamental components of voltage and current.

4. Sequence Reactances

4.1 Positive-Sequence Reactance

Comparing equations (12) and (13)

$$V_p = \sqrt{2} e^{j\phi} P_1 V_{fd} + P_2 I_p + P_3 e^{j2\phi} I_p^* \tag{14}$$

But I_p could be expressed as

$$I_p = |I_p| e^{j\theta p} \tag{15}$$

Where: θ_p is the phase angle of positive-sequence current. Equation (14) becomes

$$\begin{aligned} V_p &= \sqrt{2} e^{j\phi} P_1 V_{fd} + P_2 I_p + P_3 e^{j2\phi} I_p e^{-j2\phi p} \\ &= \sqrt{2} e^{j\phi} P_1 V_{fd} + (P_2 + P_3 e^{j2\phi} e^{-j2\theta p}) I_p \end{aligned} \tag{16}$$

But

ϕ = the angle between d-axis and phase-a at $t = 0$
$= \delta + \theta_p$

δ = the load angle which is defined as the angle between d-axis and resultant field.

$$V_p = e^{j\theta p} e^{j\delta} \sqrt{2} P_1 V_{fd} + (P_2 + P_3 e^{j2\delta}) I_p \tag{17}$$

$$= E_p + X_p I_p \tag{18}$$

From equation (17), it is noted that the positive sequence reactance depends on the load angle.

For $\delta = 0$,

$$X_{p1} = p_2 + P_3 \tag{19}$$

By substituting from (8) into (19), the following expression is obtained

$$X_{p1} = -p\ [l_s + \frac{6}{2} [\frac{L_{md}\, l_{fd}\, l_{kd}}{L_{md}\, l_{fd} + L_{md}\, l_{kd} + l_{fd}\, l_{kd}}] = -p\, L_d'' \tag{20}$$

Under steady state, for a positive-sequence current, there exists neither damper-winding current nor a.c. field-circuit current and, therefore, equation (20) becomes

$$X_{p1} = -p\ (l_s + \frac{6}{2} L_{md}) = -pL_d \tag{21}$$

The above expression is the d-axis reactance of the synchronous machine. This result is expected since the load angle is zero, and therefore the reactance of the machine is equal to that in the d-axis.

Considering the other extreme when $\delta = \pi/2$, the positive-sequence reactance becomes

$$X_{p2} = -p\ (l_s + \frac{6}{2} \frac{L_{mq}\, l_{kq}}{l_{kq} + L_{mq}}) = -p\, L_q'' \tag{22}$$

Under steady state, equation (22) becomes

$$X_{p2} = -p\ (L_s + \frac{6}{2} L_{mq}) = -pL_q \tag{23}$$

Since $\delta = 90°$, the flux is in the q-axis and therefore the positive sequence reactance is the q-axis reactance.

4.2 Negative-Sequence Reactance

Referring to equations (12, 13), one concludes that

$$V_n = P_2 \, I_n$$

But

$$V_n = X_n \, I_n$$

$$X_n = P_2 \qquad (24)$$

By substituting into (24) from (8), X_n is given by

$$X_n = -\frac{p}{2}\,(L_d'' + L_q'') = -\,pL_{n1} \qquad (25)$$

From the foregoing, it is evident that the negative-sequence inductance is equal to the arithmatic mean of L_d'' and L_q''. This is obtained since the terminal current is assumed to be the input variable in equation (7). However, if the voltage is assumed to be the input then it could be show that

$$L_{n2} = \frac{2\,L_d''\,L_q''}{L_d'' + L_q''} \qquad (26)$$

However, in practice L_d'' and L_q'' are nearly equal and therefore

$$L_n \simeq L_{n2} \simeq L_{n1} = \frac{L_d'' + L_q''}{2}$$

$$V_n = -\,pL_n \, I_n \qquad (27)$$

It is to be noted here that the negative-sequence reactance is the same under steady-state, transient or subtransient conditions. This is due to the fact that rotor circuits are not relatively at rest with respect to the field of the negative-sequence currents; thus they result in a.c. currents flowing in the field and damper circuits.

4.3 Zero-Sequence Reactance

From equation (6)

$$v_1 = -\,Pl_s \, i_1$$

or, in phasor form,

$$V_1 = -\,P\,l_s\,I_1 = -\,pl_o\,I_1 \qquad (28)$$

$$V_3 = -\,pl_o\,I_3 \qquad (29)$$

$$V_4 = -\,pl_o\,I_4 \qquad (30)$$

$$V_5 = -\,pl_o\,I_5 \qquad (31)$$

5. Conclusion

Making use of the symmetrical components approach, it has been shown that the 6-phase voltages (or currents) could be converted into instantaneous symmetrical components. However, instantaneous symmetrical quantities are of limited use and can not be directly correlated to the electrical phenomena in the machine. Therefore, they were used to develop phasor symmetrical components in which no mutual coupling appears between the various components. This result could be used as a basis for steady state short-circuit analysis of a 6-phase salient-pole synchronous machine[11].

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Appendix

A. Inductance Relations of a 6-Phase Salient-Pole Machine

The windings arrangement of a 6-phase salient-pole machine is illustrated in Fig. 1. The electrical angle α_s between the two successive stator phases is then $2\pi/6$. The direct-axis is assigned to be the zero rotor reference position, whereas the magnetic axis of phase a is assigned to be the zero stator reference position. The remaining axes of stator phases are numbered successively in a counterclockwise direction; also the q-axis leads the d-axis in the direction of rotation by 90° electrical degrees.

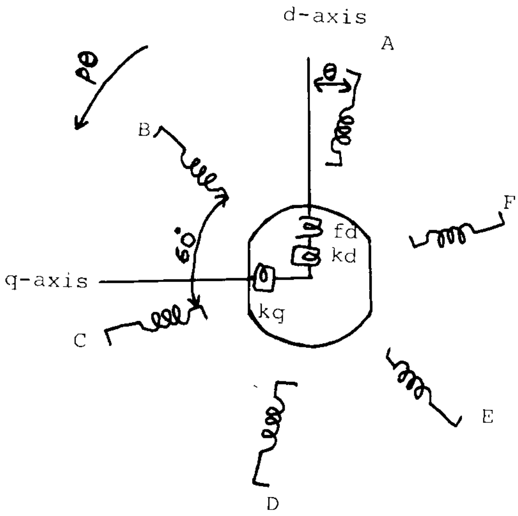

FIG. 1. Windings arrangement of a 6-phase salient-pole machine.

With the consideration of only the fundamental air-gap flux density components, expressions of the various inductances are obtained as follows.

A.1. Stator Self Inductances

The self inductances of the 6-stator phases are given as

$$\left.\begin{aligned} L_{AA} &= l_s + L_1 + L_2 \cos 2\theta \\ L_{BB} &= l_s + L_1 + L_2 \cos 2\,(\theta - 60^\circ) \\ L_{CC} &= l_s + L_1 + L_2 \cos 2\,(\theta - 120^\circ) \\ L_{DD} &= l_s + L_1 + L_2 \cos 2\,(\theta - 180^\circ) = L_{AA} \\ L_{EE} &= l_s + L_1 + L_2 \cos 2\,(\theta - 240^\circ) = L_{BB} \\ L_{FF} &= l_s + L_1 + L_2 \cos 2\,(\theta - 300^\circ) = L_{CC} \end{aligned}\right\} \qquad (A-1)$$

where

l_s = the leakage inductance of a stator phase.

L_1 = the constant component of the magnetizing inductance of a stator phase.

L_2 = the amplitude of the second harmonic component of the stator phase inductance.

A.2. Stator Mutual Inductances

The mutual inductances between each two of the stator phases are

$$L_{AB} = \frac{1}{2} L_1 + L_2 \cos (2\theta - 60^\circ) = L_{DE}$$

$$L_{AC} = -\frac{1}{2} L_1 + L_2 \cos (2\theta - 120^\circ) = L_{DF}$$

$$
\begin{aligned}
L_{AD} &= -L_1 + L_2 \cos(2\theta - 180°) \\
L_{AE} &= -\frac{1}{2} L_1 + L_2 \cos(2\theta - 240°) = L_{DB} = -L_{AB} = -L_{DE} \\
L_{AF} &= \frac{1}{2} L_1 + L_2 \cos(2\theta - 300°) = L_{DC} = -L_{AC} = -L_{DF} \\
L_{BC} &= \frac{1}{2} L_1 + L_2 \cos(2\theta - 180°) = L_{EF} \\
L_{BE} &= -L_1 + L_2 \cos(2\theta - 300°) \\
L_{BF} &= -\frac{1}{2} L_1 + L_2 \cos(2\theta - 360°) = L_{CE} = -L_{BC} = -L_{EF} \\
L_{CF} &= -L_1 + L_2 \cos(2\theta - 60°)
\end{aligned}
\tag{A-2}
$$

A.3. Rotor Self Inductances

To simplify the mathematical form of the machine inductance matrix, all the windings will be referred to one of the stator windings. Therefore, the self inductances of the field and damper windings are given as

$$
\left.
\begin{aligned}
L_{fdfd} &= l_{fd} + (L_1 + L_2) \\
L_{kdkd} &= l_{kd} + (L_1 + L_2) \\
L_{kqkq} &= l_{kq} + (L_1 - L_2)
\end{aligned}
\right\}
\tag{A-3}
$$

where

l_{fd} = the leakage inductance of the field winding referred to the stator.

l_{kd} = the leakage inductance of the d-axis damper winding referred to stator.

l_{kq} = the leakage inductance of the q-axis damper winding referred to stator.

A.4. Rotor Mutual Inductance

The mutual inductances between each two of the rotor windings referred to the stator are

$$
\left.
\begin{aligned}
L_{fdkd} &= L_{kdfd} = L_1 + L_2 \\
L_{fdkq} &= L_{kqfd} = L_{kqkd} = L_{kdkq} = 0
\end{aligned}
\right\}
\tag{A-4}
$$

A.5. Stator-to-Rotor Mutual Inductances

The mutual inductances between stator windings and field winding (referred to the stator) are

$$
\left.
\begin{aligned}
L_{Afd} &= (L_1 + L_2) \cos\theta \\
L_{Bfd} &= (L_1 + L_2) \cos(\theta - 60°) \\
L_{Cfd} &= (L_1 + L_2) \cos(\theta - 120°) \\
L_{Dfd} &= (L_1 + L_2) \cos(\theta - 180°) = -L_{Afd} \\
L_{Efd} &= (L_1 + L_2) \cos(\theta - 240°) = -L_{Bfd} \\
L_{Ffd} &= (L_1 + L_2) \cos(\theta - 300°) = -L_{Cfd}
\end{aligned}
\right\}
\tag{A-5}
$$

Similarly, the mutual inductances between the stator windings and d-axis damper winding may be written as

$$
\begin{aligned}
L_{Akd} &= (L_1 + L_2) \cos \theta \\
L_{Bkd} &= (L_1 + L_2) \cos (\theta - 60°) \\
L_{Ckd} &= (L_1 + L_2) \cos (\theta - 120°) \\
L_{Dkd} &= (L_1 + L_2) \cos (\theta - 180°) = -L_{Akd} \\
L_{Ekd} &= (L_1 + L_2) \cos (\theta - 240°) = -L_{Bkd} \\
L_{Fkd} &= (L_1 + L_2) \cos (\theta - 300°) = -L_{Ckd}
\end{aligned} \qquad (A-6)
$$

And finally, the mutual inductances between stator windings and q-axis damper winding are given by

$$
\begin{aligned}
L_{Akq} &= (L_1 - L_2) \sin \theta \\
L_{Bkq} &= (L_1 - L_2) \sin (\theta - 60°) \\
L_{Ckq} &= (L_1 - L_2) \sin (\theta - 120°) \\
L_{Dkq} &= (L_1 - L_2) \sin (\theta - 180°) = -L_{Akq} \\
L_{Ekq} &= (L_1 - L_2) \sin (\theta - 240°) = -L_{Bkq} \\
L_{Fkq} &= (L_1 - L_2) \sin (\theta - 300°) = -L_{Ckq}
\end{aligned} \qquad (A-7)
$$

تمثيـــل المـاكينــة ذات الستــة أوجــه باستخــدام المركبات المتمــاثلة

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جـــدة - المملكة العربية السعودية .

المستخلص . يقدم هذا البحث نموذجًا لمـاكينة ذات ستة أوجه ، وهذا النموذج مبني على نظرية المركبات المتماثلة . وقد طور نموذج رياضي للمـاكينة إلى مركبات متماثلة لحظية ، واستخدمت بدورها لاستنتاج معادلات تربط مركبات المتجهات المتماثلة للجهود الكهربائية والتيارات . إن هذا النموذج مفيد جدًا في التعامل مع حالات القصر غير المتماثل .